4 Examples

In this chapter the position analysis of a few selected kinematic systems shall demonstrate the crucial capabilities of the presented computer program. Moreover, it has to be pointed out that these examples represent only a fraction of the kinematic systems which can be analyzed by this computer program.

The supported basic elements of kinematic chains allow to model an arbitrarily complex kinematic system. Especially interesting is the *User defined* joint which can be used in conjunction with the special control elements for the simulation of an arbitrary relative motion between two connected bodies. The constraint elements, on the other hand, can simplify the formulation and analysis of kinematic systems significantly. Beside the joints and constraint elements, the special control elements can be used to prescribe values of the variable parameters of joints as well as constraint elements. Moreover, by using the control elements electronic or mechanical controls can be simulated.

The numerical algorithms which were used in the introduced program in general proved to be very reliable, efficient and accurate. The computations in the following examples have been executed with the precision threshold $\varepsilon=10^{-12}$.

The results of the position analysis are shown in diagrams as curves representing various input/output relations and position coordinates of points fixed in bodies. These diagrams were generated by the program automatically. Note, that the diagrams are also graphic elements appearing in the topology window, however, in these examples they are shown separately because of the space limitations.

Additionally, each example in this chapter is accompanied with the topological representation of its kinematic chain as it appears in the topology window showing the type of the used elements and their connections.

4.1 Single Loop Mechanisms

4.1.1 RPHC2R Linkage

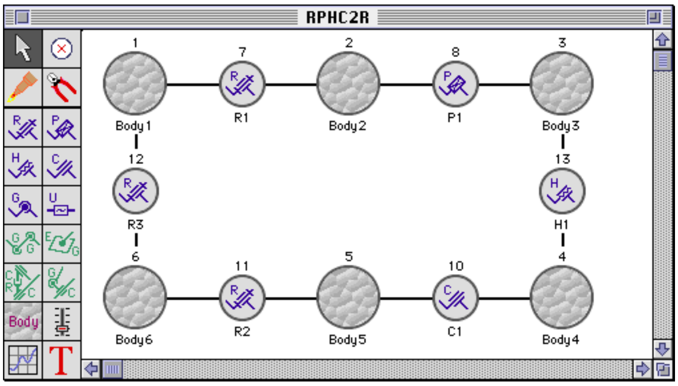

Fig 4–1: RPHC2R linkage.

Joint	Type	a	alpha	s	theta	pitch
1	R	1	30°	1	---	---
2	P	0.8	50°	---	30°	---
3	H	0.5	90°	0	---	0.2
4	C	1	-30°	---	---	---
5	R	1	90°	0	---	---
6	R	1	60°	0.5	---	---

Table 1: Denavit-Hartengerg Parameters of the RPHC2R linkage

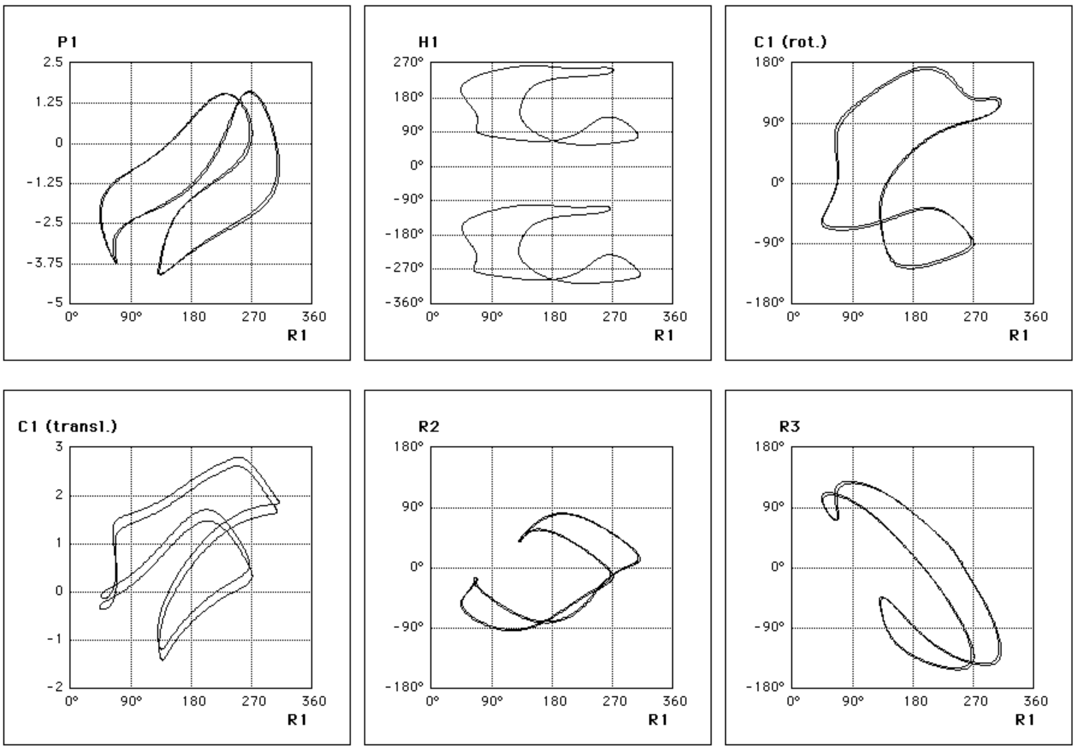

Fig 4–2: Input / Output relations of the RPHC2R Linkage.

4.2 3RC2R Linkage

This example has been found in the reference [3].

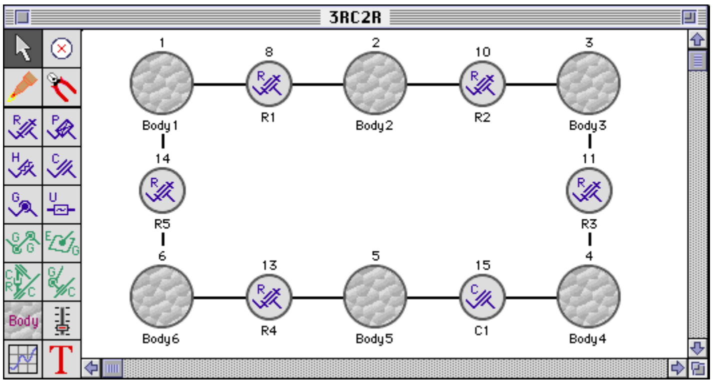

Fig 4–3: 3RC2R linkage.

Joint	Type	a	alpha	s
1	R	2	90°	8
2	R	0	90°	0
3	R	8	90°	0
4	C	2	90°	---
5	R	2	90°	2
6	R	2	90°	2

Table 2: Denavit-Hartengerg Parameters of the 3RC2R linkage

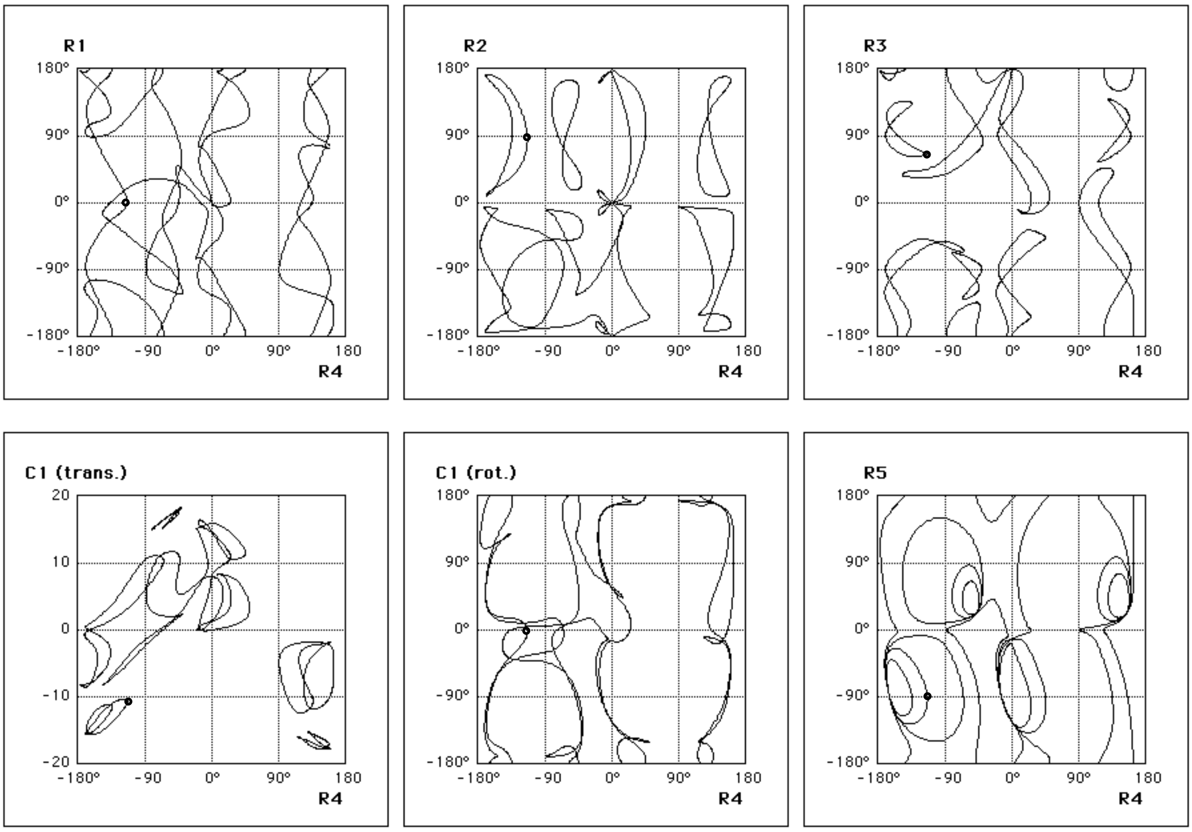

Fig 4–4: Input / output relations of the 3RC2R linkage.

4.3 Stewart Platform

In this example, the platform and the base are equilateral triangles with the side length a=1. The hydraulic cylinders are simulated with the help of six global-global constraint elements each prescribing the distance between two points in the base and the platform, respectively.

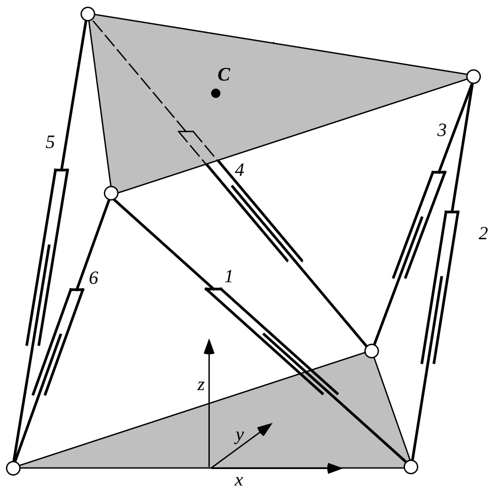

Fig 4–5: Stewart Platform

The length of the *i-th* G-G constraint element (Cylinder) is in this example defined by the parametric function

$$l_i/a = 1 + \sin[2\pi(p + i/12)]/4$$

The variable parameter *p* is supplied by the control element and used in all six equations defining the length of each cylinder. In this way the functions describing the lengths are coupled and the platform has only one degree of freedom. Therefore, for each value of the parameter p we obtain a unique position and orientation of the platform.

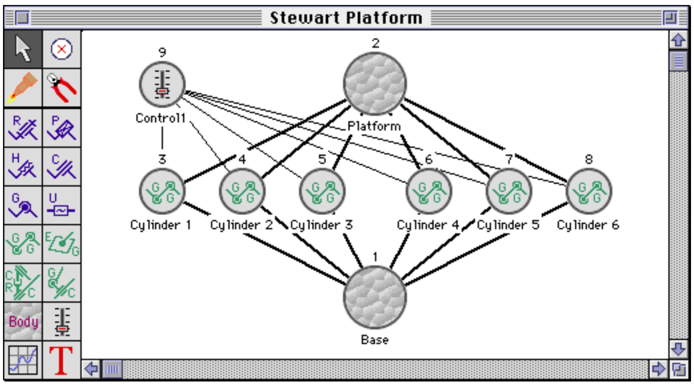

Fig 4–6: Topological representation of the Stewart platform. Note the usage of the control element supplying the variable parameter *p* used in the definition of the cylinder

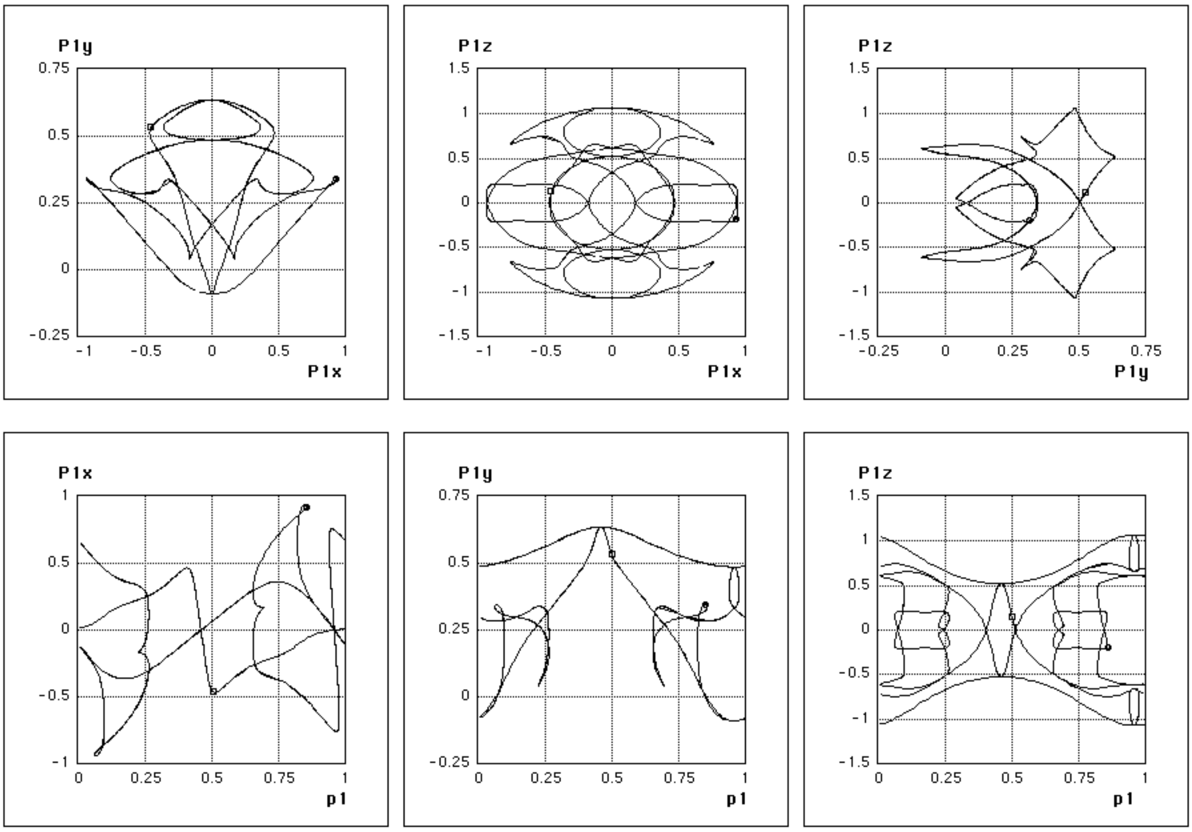

Fig 4–7: The first three diagrams show the position coordinates of the centerpoint of the platform in all three planes which are in turn fixed to the coordinate system of the base. The last three diagrams show each coordinate of the centerpoint (relative to the base) with respect to the value of the parameter *p1* which is supplied by the control element.

4.4 Examples with User Defined Joints

4.4.1 An Example of a Manipulator of the Type PUMA.

The presented robot manipulator consists of three elbows and its gripper can be rotated about three axes all perpendicular to each other (see fig. 4–8). In this example, the trajectory of the gripper has been prescribed and the corresponding variable parameters have to be found (inverse position analysis).

The origins of the coordinate systems $\underline{\mathbf{e}}^{(0)}$ and $\underline{\mathbf{e}}^{(1)}$ coincide.The distance between the $\underline{\mathbf{e}}^{(1)}$ and $\underline{\mathbf{e}}^{(2)}$ equals 1/2 while the distances between the origins of $\underline{\mathbf{e}}^{(2)}$ and $\underline{\mathbf{e}}^{(3)}$ as well as the origins of the $\underline{\mathbf{e}}^{(3)}$ and $\underline{\mathbf{e}}^{(4)}$ equal one. Moreover, the origins of the coordinate systems $\underline{\mathbf{e}}^{(4)}$, $\underline{\mathbf{e}}^{(5)}$ and $\underline{\mathbf{e}}^{(6)}$ coincide.

The gripper coordinate system $\underline{\mathbf{e}}^{(6)}$ has the same orientation as the base coordinate system and its origin is moved in the plane x-z, fixed to the coordinate system of the base at y=1.

The path of the gripper is in this plane defined by the following parametric functions:

$$|t| \le 0.5 \Rightarrow x = 0.2(1+\cos(\pi t));\ t > 0.5 \Rightarrow x = 0.7 - t;\ t < -0.5 \Rightarrow x = 0.2;$$
$$|t| \le 0.5 \Rightarrow z = 0.2(1+\sin(\pi t));\ t > 0.5 \Rightarrow z = 0.4;\ t < -0.5 \Rightarrow z = 0.5 + t;$$
$$y = 1;$$

The parameter t supplied by the control element is limited to the interval $t \in [-1, 2]$.

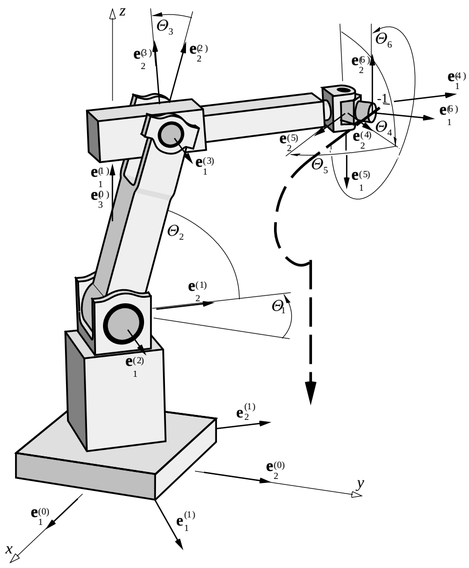

Fig 4–8: PUMA manipulator.

The trajectory of the gripper has been defined with the help of a user defined joint connecting the gripper and the base. In this manner, we obtained a closed single-loop kinematic chain as shown in the topological representation in figure 4–9.

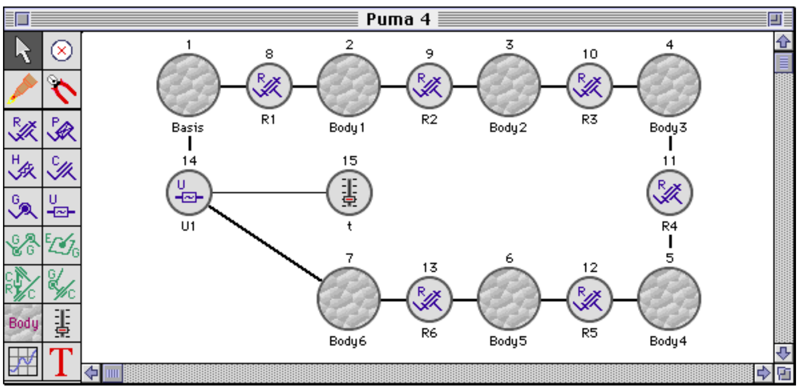

Fig 4–9: Topological representation of the manipulator. It is interpreted as a closed kinematic chain which contains a special user defined joint prescribing the path in the x-z plane.

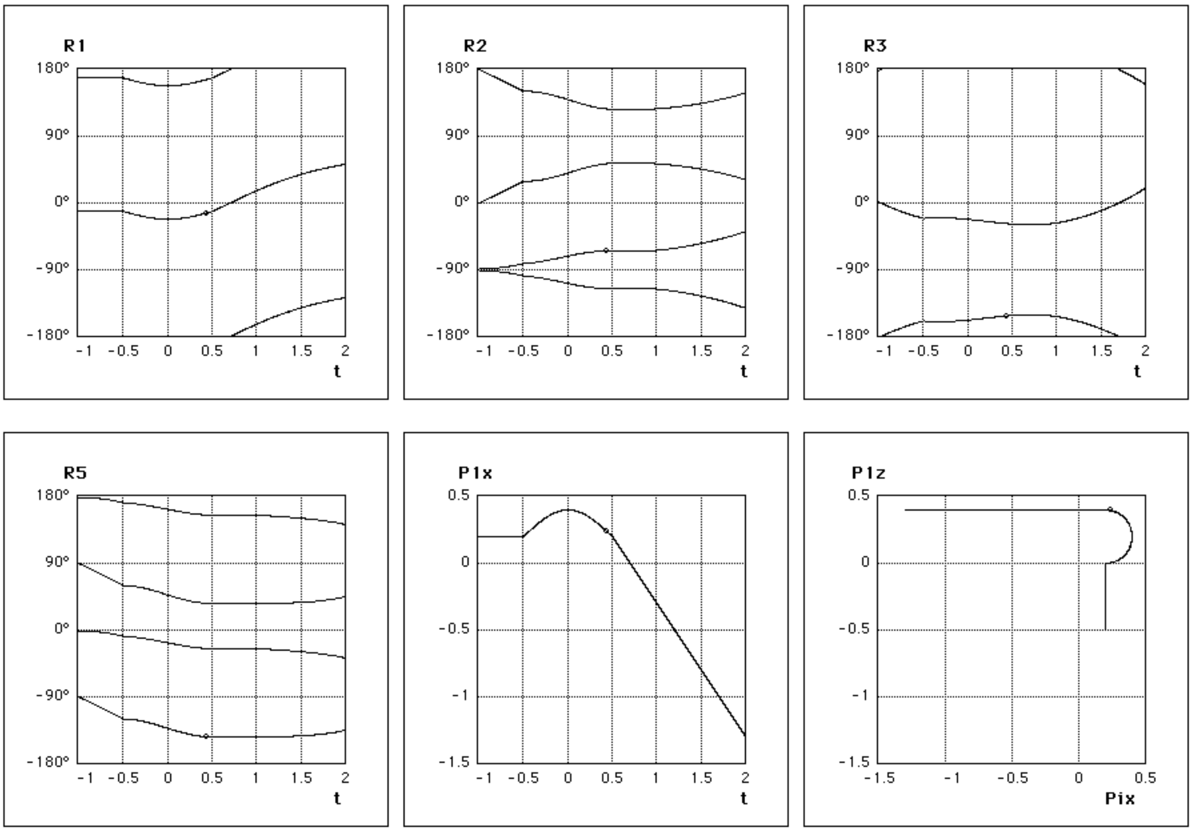

Fig 4–10: Input / Output relations of the PUMA manipulator. The last diagram shows the path of the origin of the gripper coordinate system in the x-z plane.

4.4.2 Stewart Platform with Prescribed Motion

In this example, the base and the platform are equilateral triangles with the side lengths *a* and *b*, respectively. The ratio of the side lengths is given by *a/b=4*. The cylinders are, like in the earlier example of a stewart platform, simulated with the help of six global-global constraint elements with variable length l_i which is limited to the interval $l_i \in [a, 1.5a]$.

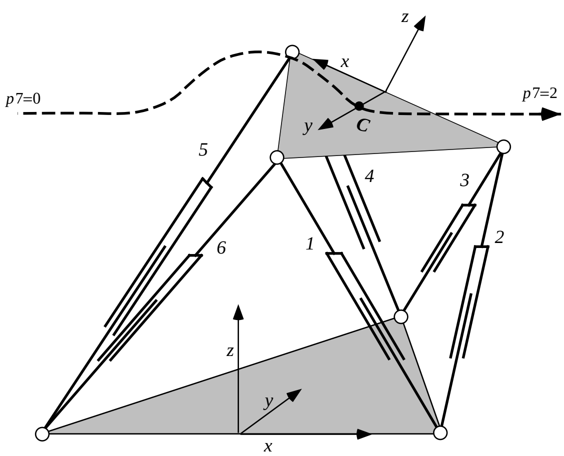

Fig 4–11: Stewart Platform.

In this example, the center point of the platform moves along the prescribed path in the x-z plane fixed in the base at $y=0.4$. Its trajectory is determined by the parametric equations with the variable parameter $0 \le p7 \le 2$ as follows:

$x=0.25p7$; $y=0.285$;

$0 \le p7 < 0.5 \Rightarrow z=1.05$; $0.5 \le p7 \le 1.5 \Rightarrow z=1.05+0.5(1-\cos(2\pi(p7-0.5)))$; $1.5 < p7 \le 2 \Rightarrow z=10.5$;

Additionally, the x axis of the coordinate system of the platform is always parallel to the tangent of the trajectory (the platform is appropriately rotated about the y axis if $0.5 \le p7 \le 1.5$).

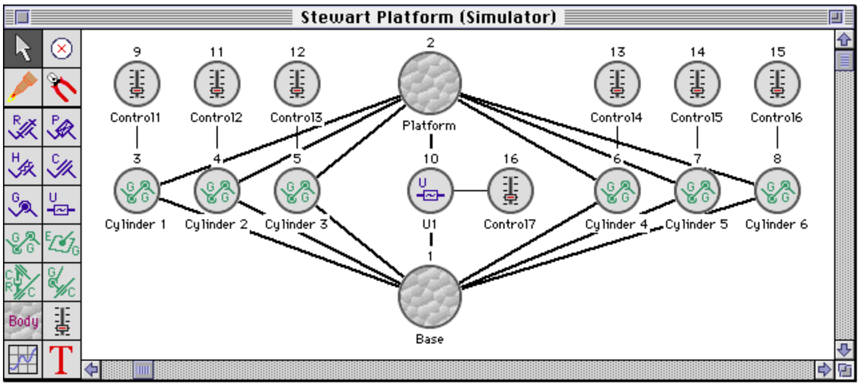

Fig 4–12: Topological representation of the Stewart platform. The parameter p7 determines the current position and orientation of the centerpoint.

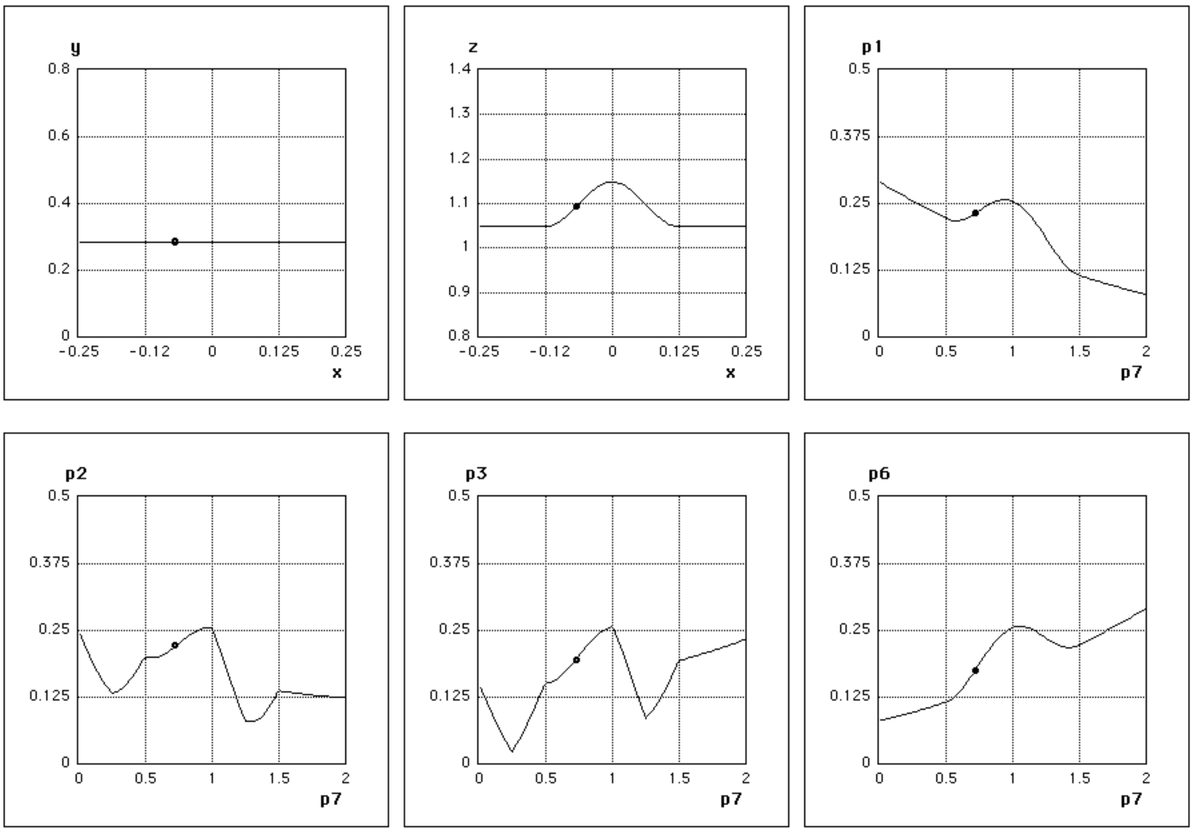

Fig 4–13: The first two diagrams show the trajectory of the centerpoint.

4.5 Multiple Loop Mechanisms

4.5.1 The Cage-Cube Mechanism

The mechanism in this example consists of twelve rectangular bodies with the side length ratio $b/a=4$ and eight eqilateral triangular plates.

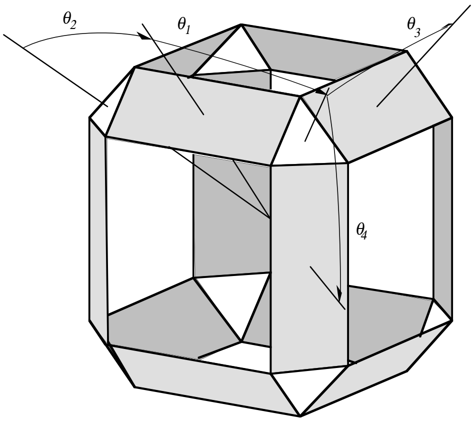

Fig 4–14: The Cage-Cube mechanism with the eight equilateral triangular and 12 rectangular bodies with the side length ratio *b/a*=4.

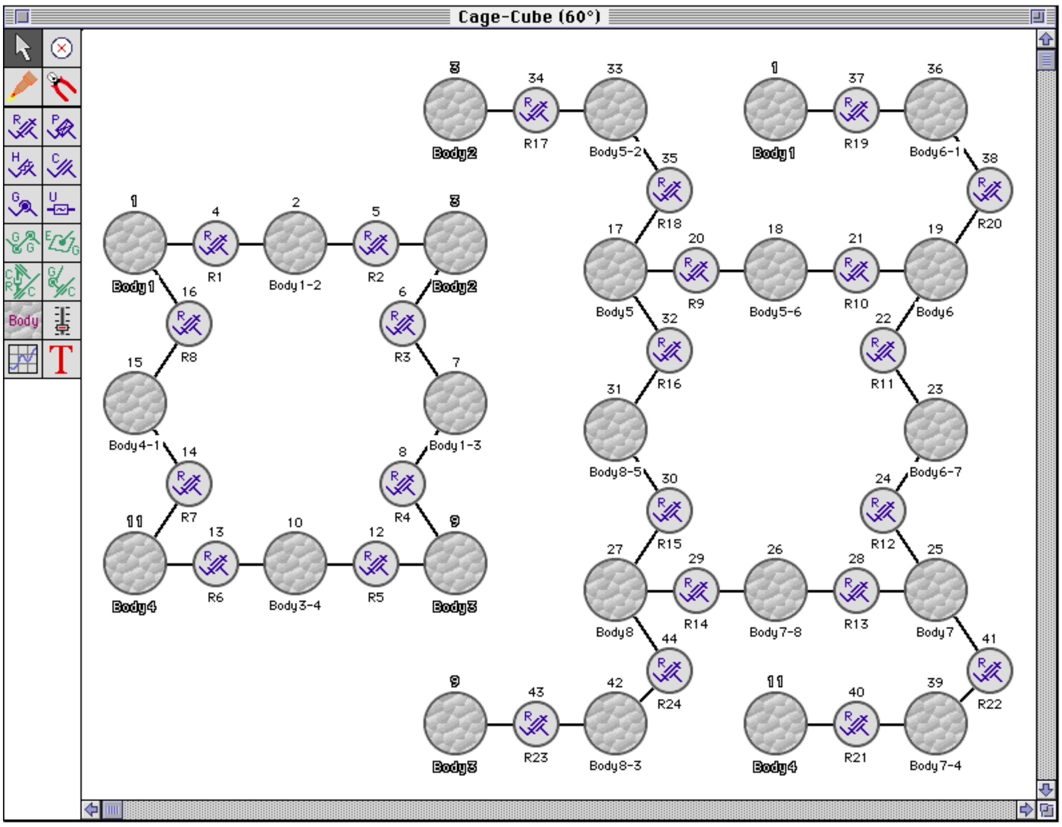

Fig 4–15: The topological representation of the Cage-Cube mechanism. Note the usage of the aliases (Body 1, Body2, Body3, Body4).

Because of the many elements (24 joints) there are several possible assembly modes. In the following input/output relations diagrams only a fraction of all existing relations (closure modes) is shown.

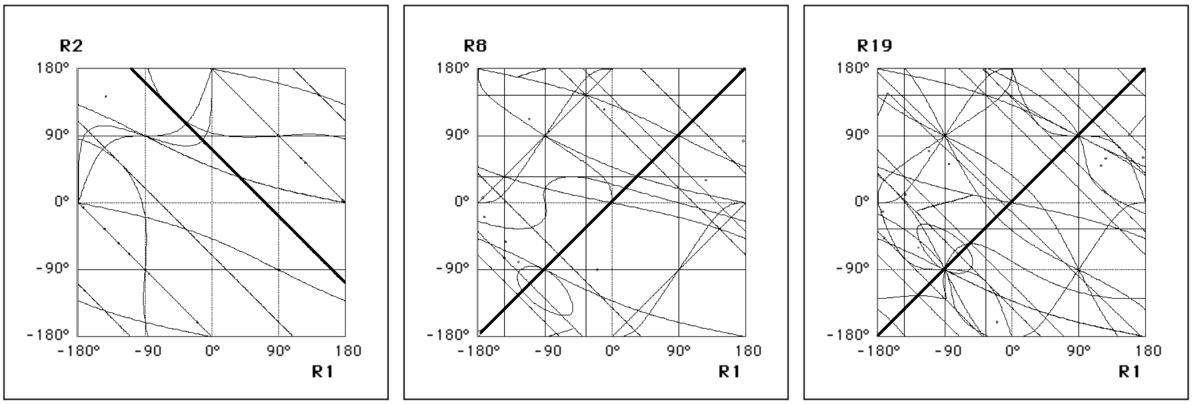

Fig 4–16: Input / output relations of the Cage-Cube mechanism.

4.5.2 Spatial Straight- Line Linkage

This straight-line linkage consists of one planar four-bar linkage (parallelogram with the side length ratio *b/a*=1.5) and two spherical four bar linkages [5]. Both spherical linkages have the same dimensions (two angles between the axes of rotation equal α=30° and the remaining two angles equal β=90°). The points C_1 and C_2 move along straight lines relative to the base body if the input angle θ_1 of the planar four-bar linkage is changed.

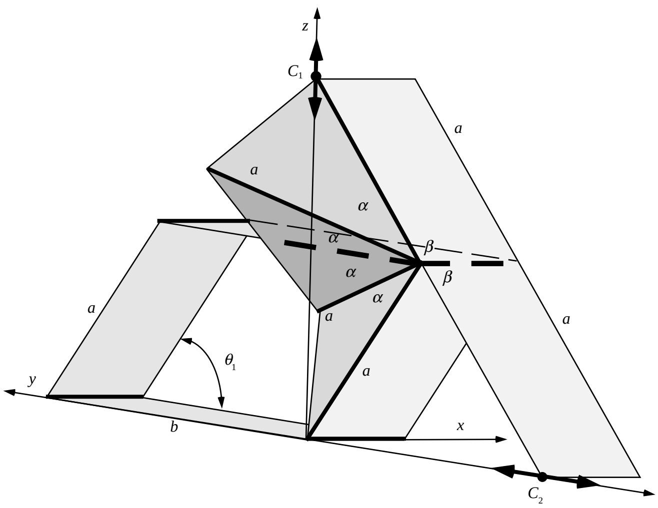

Fig 4–17: Spatial straight-line linkage. The thick lines represent the axes of revolute joints.

This spatial kinematic chain contains three loops. One of them corresponds to the planar four bar linkage while the remaining two correspond to the spherical linkages.

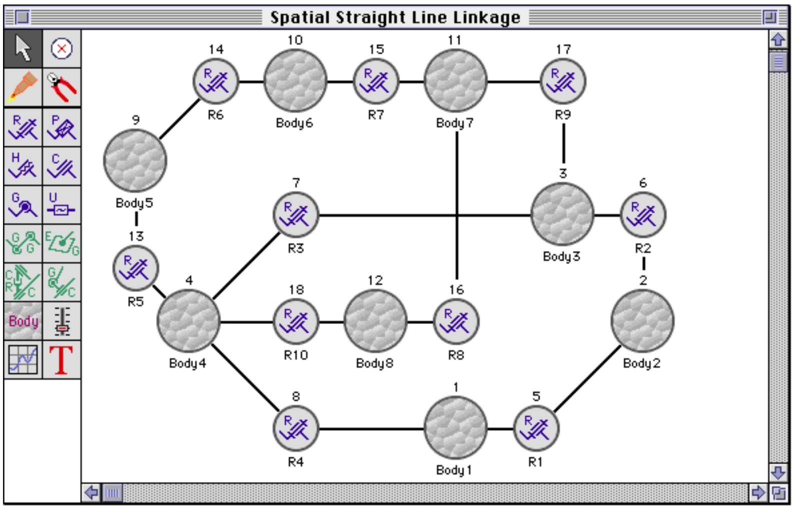

Fig 4–18: Topological representation of the spatial hybrid straight-line linkage.

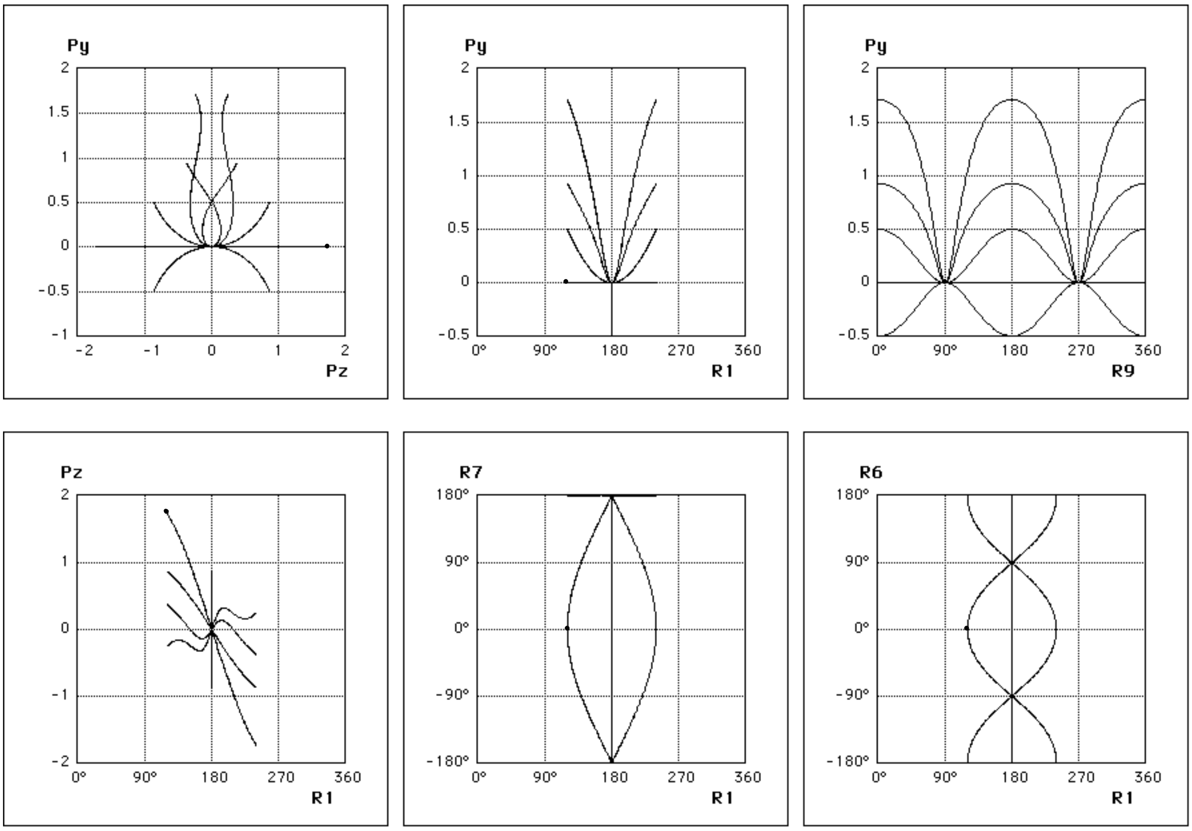

Fig 4–19: The first diagram shows the path of the point C_1 in the y-z plane. There are several closure modes, however, only two of them represent straight-line linkages.